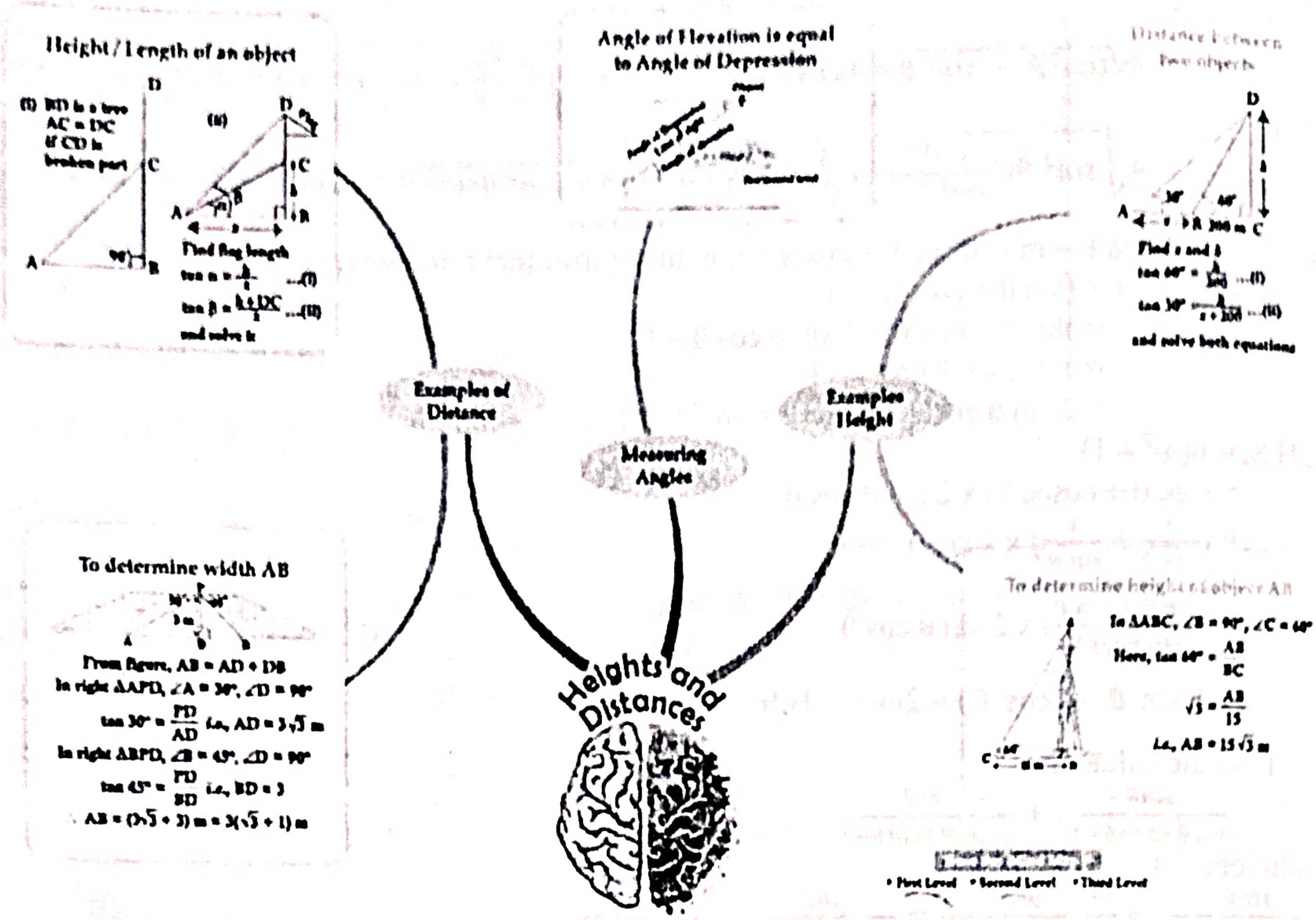
CHAPTER 9
APPLICATIONS OF TRIGONOMTERY
MIND MAP



GIST OF THE CHAPTER

1.Line of sight
2.Angle of elevation
3.Angle of depression

DEFINITION

1.The line of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer.

2.The angle of elevation of an object viewed, is the angle formed by the line of sight with the horizontal when it is above the horizontal level, i.e., the case when we raise our head to look at the object.

3.The angle of depression of an object viewed, is the angle formed by the line of sight with the horizontal when it is below the horizontal level, i.e., the case when we lower our head to look at the object.

MULTIPLE CHOICE QUESTIONS(1 MARK QUESTIONS)

1.If the length of the shadow of a tree decreases then the angle of elevation :

(a) Increases (b) Decreases (c) Remains the same (d) None of the above

Ans (a) Increases

2.The angle of elevation of the top of a building from a point on the ground, which is 30 m away from the foot of the building, is 30°. The height of the building is:

(a) 10 m (b) 30/√3 m (c) √3/10 m (d) 30 m

Ans : (b) 30/√3 m

3.If the height of the building and distance from the building foot to a point is increased by 20%, then the angle of elevation on the top of the building:

(a) Increases (b) Decreases (c) Do not change (d) None of the above

Ans : (c) Do not change

4.If a tower 6m high casts a shadow of $2\sqrt{3}$ m long on the ground, then the sun's elevation is:

(a) 60° (b) 45° (c) 30° (d) 90°

Ans: (a) 60°

5. The angle of elevation of the top of a building 30 m high from the foot of another building in the same plane is 60°, and also the angle of elevation of the top of the second tower from the foot of the first tower is 30°, then the distance between the two buildings is:

(a) $10\sqrt{3}$ m (b) $15\sqrt{3}$ m (c) $12\sqrt{3}$ m (d) 36 m

Ansr: (a) $10\sqrt{3}$ m

6.The angle formed by the line of sight with the horizontal when the point is below the horizontal level is called:

(a) Angle of elevation (b) Angle of depression

(c) No such angle is formed (d) None of the above

Ans : (b) Angle of depression

7.From a point on the ground, which is 15 m away from the foot of the tower, the angle of elevation of the top of the tower is found to be 60°. The height of the tower (in m) standing straight is:

(a) $15\sqrt{3}$ (b) $10\sqrt{3}$ (c) $12\sqrt{3}$ (d) $20\sqrt{3}$

Ans: (a) $15\sqrt{3}$

8.The angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level is called:

(a) Angle of elevation (b) Angle of depression

(c) No such angle is formed (d) None

Ans: (a) Angle of elevation

9. When the shadow of a pole h metres high is $\sqrt{3}h$ metres long, the angle of elevation of the Sun is

(a) 30° (b) 60° (c) 45° (d) 15°

Ans: (a) 30°

10.A ladder makes an angle of 60° with the ground, when placed along a wall. If the foot of ladder is 8 m away from the wall, the length of ladder is

(a) 4 m (b) 8 m (c) $8\sqrt{3}$ m (d) 16 m

Ans: (d) 16 m

ASSERTION AND REASONING QUESTIONS

DIRECTION: In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

1.Assertion (A): The height of a tower can be found using the angle of elevation and the horizontal distance from the base.

Reason (R) : The relation $\tan\theta = \frac{Height}{Base}$ holds true in right-angled triangles.

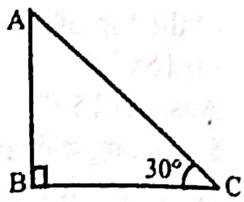
Ans:(b)

2.Assertion (A): The angle of elevation of the top of a building increases as the observer approaches the building.

Reason (R): A larger angle of elevation implies a taller object.

Ans:(c)

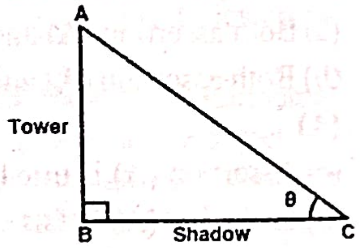
3.**Assertion (A):** When the sun is lower in the sky (early morning or evening), the shadow of a vertical object is larger.
Reason (R): The angle of elevation of the sun is small during early morning or evening.
Ans:(a)
4.**Assertion (A):** The angle of depression from a lighthouse to a boat is equal to the angle of elevation from the boat to the lighthouse.
Reason (R): These angles are alternate interior angles formed by a transversal intersecting two parallel lines.
Ans: (a)
5.**Assertion (A):** The height of a tower can be calculated by observing the angle of elevation from two different points on a straight line and knowing the distance between them.
Reason (R): The angle of elevation of the top of a tower increases as the observer approaches the tower.
Ans: (b)
6. **Assertion (A):** If the length of the shadow of a vertical pole is equal to its height, then the angle of elevation of the sun is 45°.
Reason (R): According to Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{altitude})^2 + (\text{base})^2$,
Ans: (b)
7.**Assertion (A):** In the figure, if BC = 20 m, then height AB is 11.56 m.
Reason (R): $\tan\theta = \frac{AB}{BC} = \frac{\text{perpendicular}}{\text{base}}$, where θ is the angle ∠ACB



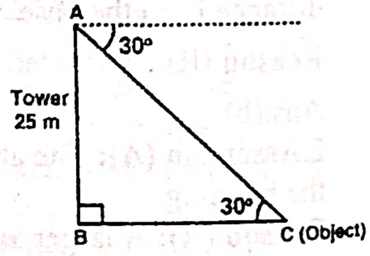
Ans: (a)
8.**Assertion (A):** The value of sin 90° = 0.
Reason (R): The value of cos 90° = 0
Ans: (d)
9.**Assertion (A):** If the shadow of a vertical pole is $\frac{1}{\sqrt{3}}$ times its height, then the angle of elevation of the tower is 60^0
Reason (*R*) : If the angle of elevation of a vertical pole is 45^0, then the shadow of the vertical pole is same as its height.
Ans: (b)
10.**Assertion (A):** $\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$
Reason (R): tan 90° = 0
Ans: (c)

VERY SHORT ANSWER TYPE QUESTIONS (2 MARK QUESTIONS)

1.If the height and length of a shadow of a tower are the same, then find the angle of elevation of Sun



Solution : Let AB be the tower and BC be its shadow.
AB = BC
In right triangle ABC, $\tan\theta = \frac{AB}{BC}$
$\tan\theta = \frac{AB}{AB}$ (since AB = BC)
$\tan\theta = 1 \Rightarrow \tan\theta = \tan 45^\circ \therefore \theta = 45^\circ$
2.The angle of depression of an object on the ground, from the top of a 25 m high tower is 30°. find the distance of the object from the base of tower

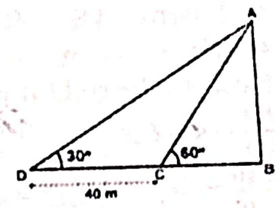


Solution: Let AB be the tower and BC be the distance of the object (at C) from the base of the tower.
In right triangle ABC,
$\tan 30^\circ = \frac{AB}{BC}$
$\frac{1}{\sqrt{3}} = \frac{25}{BC} \Rightarrow BC = 25\sqrt{3}$ m

3.The shadow of a tower standing on level ground is found to be 40 m longer when the Sun's altitude is 30° than when it was 60°. Find the height of the tower.

Solution: Let AB be h m and BC be x m. From the question, DC is 40 m longer than BC. BD = (40 + x) m

In ΔABC, $\tan 60° = \frac{AB}{BC} \Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow x = \frac{h}{\sqrt{3}}$ ....(i)



In ΔABD,

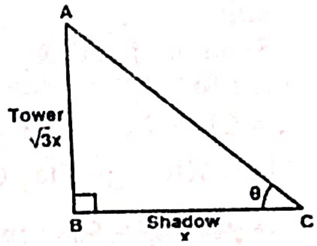
$\tan 30° = \frac{AB}{BD} = \frac{1}{\sqrt{3}}$

$\frac{h}{x+40} = \frac{1}{\sqrt{3}} \Rightarrow x + 40 = \sqrt{3}h$

$\frac{h}{\sqrt{3}} + 40 = \sqrt{3}h$ [using (i)]

$h + 40\sqrt{3} = 3h \Rightarrow h = 20\sqrt{3}m$

4.The ratio of the height of a tower and the length of its shadow on the ground is $\sqrt{3} : 1$. Find the angle of elevation of the Sun



Solution: Given, AB : BC = $\sqrt{3} : 1$

So, AB = $\sqrt{3}x$ and BC = x

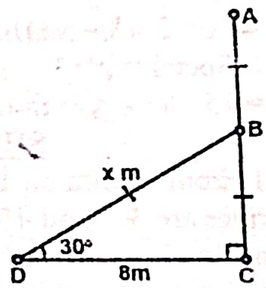
In right triangle ABC,

$\tan \theta = \frac{AB}{BC}$

$\tan \theta = \frac{\sqrt{3}x}{x} \Rightarrow \tan \theta = \sqrt{3} \Rightarrow \tan \theta = \tan 60° \Rightarrow \theta = 60°$

5.A tree breaks due to a storm and the broken part bends so that the top of the tree touches the ground making an angle of 30° with the ground. The distance between the foot of the tree to the point where the top touches the ground is 8 m. find The height of the tree.

Solution: Let AC be the initial height of the tree. Let the bent portion of the tree be AB = x m and the remaining portion BC = h m.



So, the height of the tree AC = (x + h) m

In right ΔBCD,

$\tan 30° = \frac{BC}{DC} = \frac{h}{8} \Rightarrow \frac{h}{8} = \frac{1}{\sqrt{3}} \Rightarrow h = \frac{8}{\sqrt{3}}m$

In right ΔBCD, $\cos 30° = \frac{DC}{BD}$

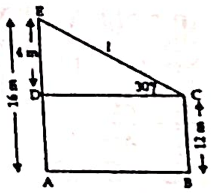
$\frac{\sqrt{3}}{2} = \frac{8}{x} \Rightarrow x = \frac{16}{\sqrt{3}}m$

So, $x + h = \frac{16}{\sqrt{3}} + \frac{8}{\sqrt{3}} = 8\sqrt{3}m$

6.The tops of two poles of height 16 m and 12 m are connected by a wire, the wire makes angle of 30° with the horizontal, find length of wire.

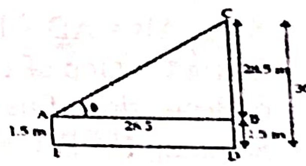
Solution: In right ΔEDC, $\sin \theta = \frac{ED}{EC}$

$\sin 30 = \frac{4}{l} \Rightarrow \frac{1}{2} = \frac{4}{l} \Rightarrow l = 8$ m



7.An observer 1.5 m tall is 28.5 m away from a tower of height 30 m. Find the angle of elevation of the top of tower from his eye.

Solution:

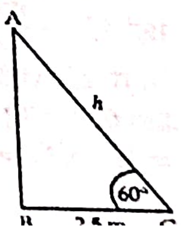


In right ΔABC, $\tan \theta = \frac{BC}{AB}$

$\tan \theta = \frac{28.5}{28.5} = 1 = \tan 45° \quad \therefore \theta = 45°$

8.A ladder, leaning against a wall, makes an angle of 60° with the horizontal. If the foot of the ladder is 2.5 m away from the wall, find the length of the ladder.

Solution:

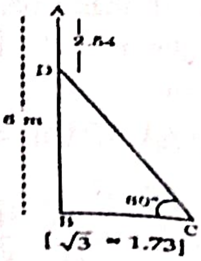


Let AC be the ladder and the foot C is 2.5 m away from the wall AB.

$\cos 60° = \frac{BC}{AC} = \frac{2.5}{h}$

$\frac{1}{2} = \frac{2.5}{h} \quad \therefore h = 5$ m.

9.AB is a 6 m high pole and CD is a ladder inclined at an angle of 60° to the horizontal and reaches up to a point D of pole. If AD = 2.54 m, find the length of the ladder. (use $\sqrt{3} = 1.73$)



Solution:

AB = AD + DB = 6 m (given)

$\Rightarrow$ 2.54 m + DB = 6 m

$\Rightarrow$ DB = 3.46 m

Now, in the right Δ BCD.

$\frac{BD}{CD} = \sin 60$

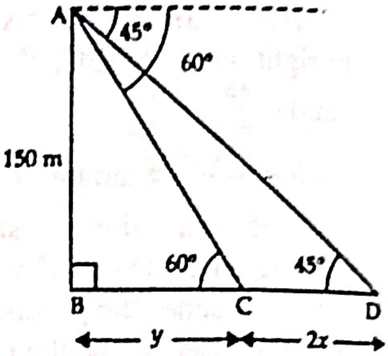
$\frac{3.46}{CD} = \frac{\sqrt{3}}{2}$ $\therefore$ CD = 4 m

10.A moving boat is observed from the top of a 150 m high cliff moving away from the cliff. The angle of depression of the boat changes from 60° to 45° in 2 minutes. Find the speed of the boat in m/h.

Solution: Let C & D be two positions of the boat & AB be the cliff & let speed of boat be x m/ min.

Let BC = y

$\therefore$ CD = 2x ($\because$ Distance = speed × time)



In ΔABC, $\frac{150}{y} = \tan 60°$

$\Rightarrow y = \frac{150}{\sqrt{3}} = 50\sqrt{3}$ m

In ΔABD, $\frac{150}{y+2x} = \tan 45° \Rightarrow \frac{150}{50\sqrt{3}+2x} = 1$

$50\sqrt{3} + 2x = 150$

$\therefore x = \frac{150 - 50\sqrt{3}}{2} = 75 - 25\sqrt{3}$

$\Rightarrow x = 25(3 - \sqrt{3})$

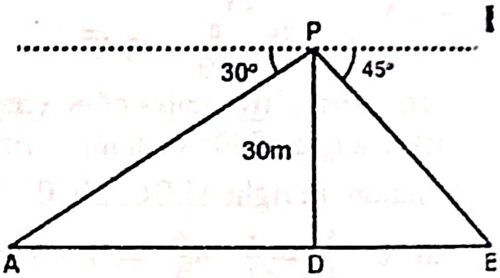
$\therefore$ Speed = $25(3 - \sqrt{3})$ m/min

= $1500(3 - \sqrt{3})$ m/hr.

SHORT ANSWER TYPE QUESTIONS (3 MARK QUESTIONS)

1.From a point on a bridge across a river the angle of depression of the banks on opposite sides of the river are 30° and 45° respectively. If the bridge is at the height of 30 m from the banks, find the width of the river.

Solution: Let, A and B represent the points on the bank on opposite sides of the river. And, AB is the width of the river.



AB = AD + DB

In right ΔAPD

So, $\tan 30° = \frac{PD}{AD}$

$\frac{1}{\sqrt{3}} = \frac{PD}{AD}$

AD = $30\sqrt{3}$ m

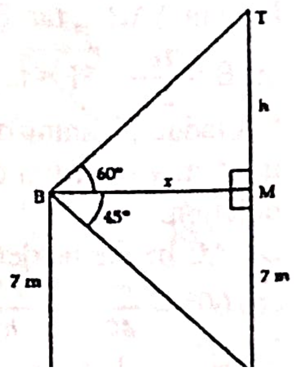
In right ΔPBD, $\tan 45° = \frac{PD}{BD}$ $\Rightarrow$ BD = 30 m

Since AB = AD + DB = $30\sqrt{3} + 30 = 30(\sqrt{3} + 1)$ m

2.From the top of a 7 m high building, the angle of elevation of the top of a tower is 60° and the angle of depression of its foot is 45°. Find the height of the tower.

Solution: Let BG be building

TW be Tower, then: BM = x, ∠MBT = 60° ∠MBW = 45°



In rt. ΔBMW

$\tan 45° = \frac{WM}{BM}$ $\Rightarrow 1 = \frac{7}{x} \Rightarrow x = 7$ m

In rt. ΔTMB

$\tan 60° = TM/BM \Rightarrow \sqrt{3} = h/x$

$\Rightarrow h = \sqrt{3}x = 7\sqrt{3}$

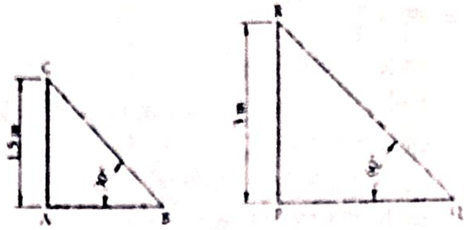
Height of Tower = TW = TM + MW

$= (7\sqrt{3} + 7)\text{m} = 7(\sqrt{3} + 1)\text{m}$

3.A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of 30° to the ground, whereas for elder children, she wants to have a steep slide at a height of 3 m, and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?

Solution:(i) In ΔABC, sin 30° = AC / BC

$\frac{1}{2} = \frac{1.5}{BC}$, BC = 1.5 × 2 ∴ BC = 3



(ii) In ΔPRQ, $\sin Q = \frac{PR}{QR} \Rightarrow \sin 60° = \frac{3}{QR} = \frac{\sqrt{3}}{2} = \frac{3}{QR}$

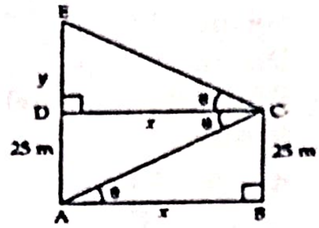
$\Rightarrow QR = \frac{6}{\sqrt{3}} = 2\sqrt{3}$ m

Length of the slide for children below 5 years = 3 m

Length of the slide for elder children = $2\sqrt{3}$ m

4.From the top of a 25 m high cliff, the angle of elevation of a tower is found to be equal to the angle of depression of the foot of the tower. Find height of the tower.

Solution: In right ΔABC,



$\tan\theta = \frac{BC}{AB} \Rightarrow \tan\theta = \frac{25}{x}$

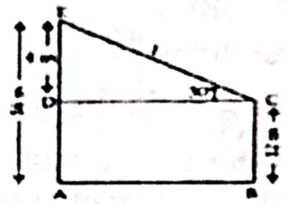
In Δ CDE, $\tan\theta = \frac{DE}{CD}$

$\Rightarrow \tan\theta = \frac{y}{x}$

$\frac{25}{x} = \frac{y}{x} \Rightarrow y = 25$ m ∴ Height of tower = x + y = 25 + 25 = 50 m

5.The tops of two poles of height 16 m and 12 m are connected by a wire, the wire makes angle of 30° with the horizontal, find length of wire.

Solution: In right ΔEDC,



$\text{Sin}\,\theta = \frac{ED}{EC}$

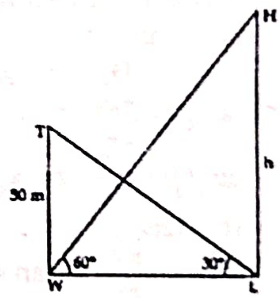
$\text{Sin}\,30 = \frac{4}{l} \Rightarrow \frac{1}{2} = \frac{4}{l} \Rightarrow l = 8$ m

6.The angle of elevation of the top of a hill at the foot of a tower is 60° and the angle of elevation of the top of the tower from the foot of the hill is 30°. If height of the tower is 50 m, find the height of the hill.

Solution: Let HL be Hill and TW be Tower angle of elevations ∠WLT = 30°

∠LWH = 60°, let WL = x

In rt. ΔLWT



$\Rightarrow \frac{50}{x} = \tan 30°$

$\Rightarrow x = 50\sqrt{3}$ .... (i)

In rt. ΔWLH

$\Rightarrow \frac{h}{x} = \tan 60° \Rightarrow \frac{h}{50\sqrt{3}} = \sqrt{3} \Rightarrow h = 50\sqrt{3} \times \sqrt{3} = 150$ [Using (i)]

7.The angles of depression of the top and bottom of a 50 m high building from the top of a tower are 45° and 60° respectively. Find the height of the tower and the horizontal distance between the tower and the building, (use $\sqrt{3} = 1.73$).

Solution: Let the height of the tower AB be h metres and the horizontal distance between the tower and the building BC be x metres.

So, AE = (h – 50) m

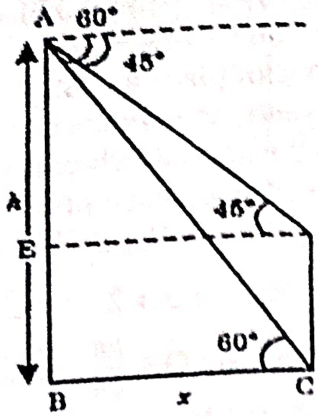
In ΔAED, $\tan 45° = \frac{AE}{ED}$

$\Rightarrow 1 = \frac{h-50}{x}$

$\Rightarrow x = h - 50$ .... (i)

In ΔABC, $\tan 60° = \frac{AB}{BC}$

$\Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow \sqrt{3}x = h$ .... (ii)



Using (i) and (ii), we get

$\Rightarrow x = \sqrt{3}x - 50$

$\Rightarrow x(\sqrt{3} - 1) = 50$

$\Rightarrow x = \frac{50}{\sqrt{3}-1}$

$\Rightarrow x = 68.25$ m

Substituting the value of x in (i), we get

$68.25 = h - 50$

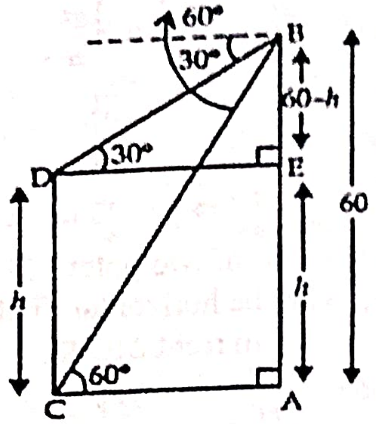
$\Rightarrow h = 68.25 + 50$

$\Rightarrow h = 118.25$ m

8.There are two poles, one each on either bank of a river, just opposite to each other. One pole is 60 m high. From the top of this pole, the angles of depression of the top and the foot of the other pole are 30° and 60° respectively. Find the width of the river and height of the other pole.

Solution: Let AB, CD be two poles separated by river of width CA with AB = 60 m and let CD = h m

Draw DE ⊥ AB



$BE = AB - EA = (60 - h)$ m

In right ΔBAC

$\tan 60 = \frac{BA}{CA}$

$\Rightarrow \sqrt{3} = \frac{60}{CA} \Rightarrow CA = \frac{60}{\sqrt{3}}$ m $= 20\sqrt{3}$ m

Thus, width of river $= 20\sqrt{3}$ m

In ΔBED

$\tan 30° = \frac{BE}{DE}$

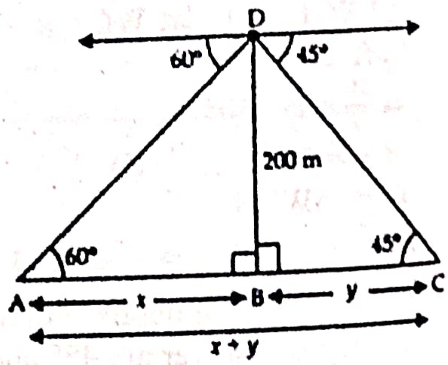
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{60-h}{20\sqrt{3}} \Rightarrow 20 = 60 - h$

$\Rightarrow h = 60 - 20 = 40$

Height of the other pole = 40 m.

9.Two ships are there in the sea on either side of a lighthouse in such a way that the ships and the lighthouse are in the same straight line. The angles of depression of two ships as observed from the top of the lighthouse are 60° and 45°. If the height of the lighthouse is 200 m, find the distance between the two ships. [Use $\sqrt{3} = 1.73$]

Solution:



In Δ ABD,

$\tan \theta = \frac{BD}{AB}$

$\tan 60 = \frac{200}{x} \Rightarrow x = \frac{200}{\sqrt{3}} = 115.4$ m

In ΔDBC,

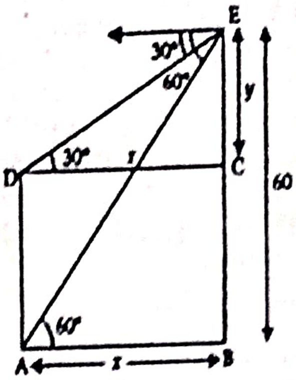
$\tan \theta = \frac{BD}{BC} \Rightarrow \tan 45 = \frac{200}{y}$

$\Rightarrow y = 200$ m

Distance between two ships = x + y

= 115.4 + 200 = 315.4 m

10.The angles of elevation and depression of the top and the bottom of a tower from the top of a building, 60 m high, are 30° and 60° respectively. Find the difference between the heights of the building and the tower and the distance between them.



Answer:

In ΔABE,

$\tan\theta = \frac{BE}{AB} \Rightarrow \tan 60 = \frac{60}{x} \Rightarrow \sqrt{3} = \frac{60}{x}$

$x = \frac{60}{\sqrt{3}} = 20\sqrt{3}$ m

In Δ DCE,

$\tan\theta = \frac{EC}{CD}$

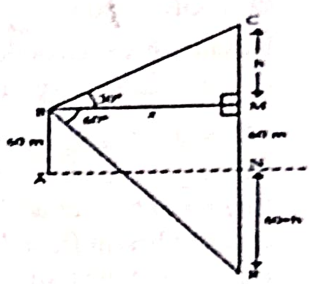
$\tan 30 = \frac{y}{x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{y}{20\sqrt{3}} \Rightarrow y = 20$ m

∴ Difference between heights of the building and tower = y = 20 m

Distance between tower and building = $x = 20\sqrt{3}$ m

LONG ANSWER TYPE QUESTIONS (5 MARK QUESTIONS)

1.The angle of elevation of a cloud from a point 60 m above the surface of the water of a lake is 30° and the angle of depression of its shadow in water of lake is 60°. Find the height of the cloud from the surface of water.



Solution: Let C be cloud &B be point 60 m above the surface of water angle of elevation of cloud = ∠MBC = 30°

Angle & Depression of clouds reflection 'R' ∠MBR = 60°

Let BM = x, CM = h, NR = 60 + h,

MR = 60 + 60 + h = 120 + h

In rt. ΔBMC

$\frac{h}{x} = \tan 30° = \frac{1}{\sqrt{3}}$

$\Rightarrow x = h\sqrt{3}$ ... (i)

In rt. ΔBMR

$\frac{60+60+h}{x} = \tan 60°$

$\Rightarrow \frac{120+h}{x} = \sqrt{3}$

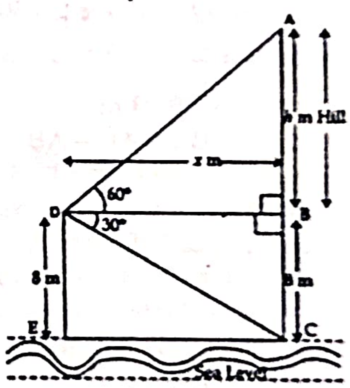
$\Rightarrow 120 + h = h\sqrt{3} \times \sqrt{3}$ [using (i)]

$\Rightarrow h = 60$

∵ height of cloud from surface of water = (60 + 60)m = 120 m.

2.A man is standing on the deck of a ship, which is 8 m above water level. He observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. Calculate the distance of the hill from the ship and the height of the hill.

Solution: Let x m be the distance between hill and man. The angles of elevation and depression are 60° and 30° respectively. Various arrangements are as shown in the figure.



In right ΔDBA, $\frac{AB}{BD} = \tan 60°$

$\frac{h}{x} = \sqrt{3}$

$h = \sqrt{3}x$ .... (i)

In right ΔDBC, $\frac{BC}{BD} = \tan 30°$

$\frac{8}{x} = \frac{1}{\sqrt{3}}$

$x = 8\sqrt{3}$ .... (ii)

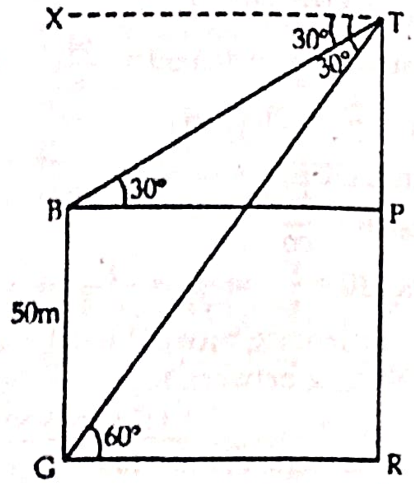
From (i) and (ii), we get

$h = \sqrt{3} \times 8\sqrt{3} = 8 \times 3 = 24$ m

∴ Height of hill = (h + 8) m = (24 + 8) m = 32 m

Hence, height of hill and distance of man from hill are 32 m and $8\sqrt{3}$ m respectively.

3.The angles of depression of the top and bottom of a building 50 metres high as observed from the top of a tower are 30° and 60°, respectively. Find the height of the tower and also the horizontal distance between the building and the tower.



Solution: In fig. Let BG be the building & TR be Tower.

∠XTB = 30°, ∠XTG = 60°

∠TBP = ∠XTB = 30° [alt. angles]

∠TGR = ∠XTG = 60° [alt. angles]

In ΔBTP ⇒ tan 30° $= \frac{TP}{BP}$

BP = TP√3

In ΔTGR tan 60° $= \frac{TR}{GR}$ ⇒ $\sqrt{3} = \frac{TR}{GR}$

Now, TP√3 $= \frac{TR}{\sqrt{3}}$ (as BP = GR)

⇒ 3TP = TP + PR

⇒ 2TP = BG ⇒ TP = 502 m = 25 m

Now, TR = TP + PR = (25 + 50) m.

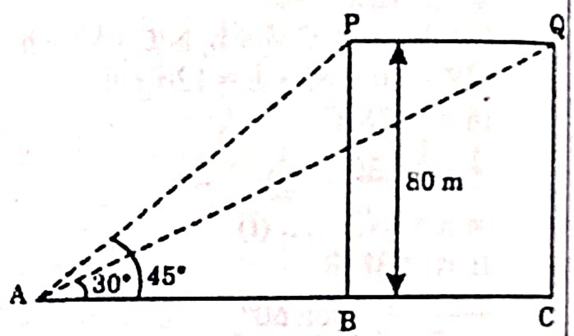
Height of tower = TR = 75 m.

In ΔTGR tan 60° $= \frac{TR}{GR}$ ⇒ $\sqrt{3} = \frac{TR}{GR}$

Distance between building and tower = GR

⇒ GR = $\frac{75}{\sqrt{3}}$ m = 25 √3 m

4.A bird is sitting on the top of a 80 m high tree. From a point on the ground, the angle of elevation of the bird is 45°. The bird flies away horizontally in such a way that it remained at a constant height from the ground. After 2 seconds, the angle of elevation of the bird from the same point is 30°. Find the speed of flying of the bird. (Take √3 =1.732)



Solution: Let P and Q be the two positions of the bird, and let A be the point of observation. Let ABC be the horizontal line through A.

Given, The angles of elevations ∠PAB = 45° and ∠QAB = 30°, respectively.

∴ ∠PAB = 45° and ∠QAB = 30°

Also, PB = 80 m

In ΔABP, we have tan 45° $= \frac{BP}{AB}$

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= 80 √3 – 80 = 80(√3 – 1)

So, the bird covers 80 (√3 – 1) m in 2 s.

Thus, speed of the bird

= Distance Time =80(√3−1)2 m/s

= 40(√3−1)×60×601000 km/h

= 144 (1.732 – 1) km/h

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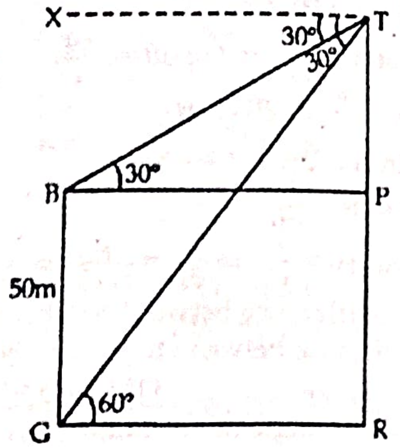

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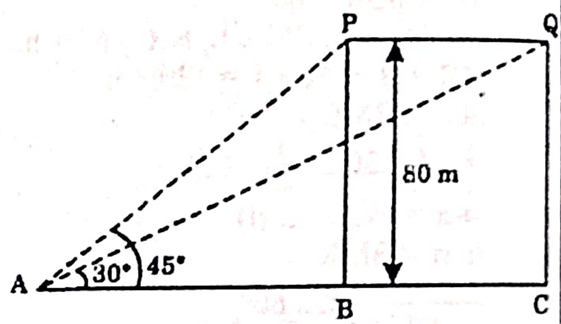

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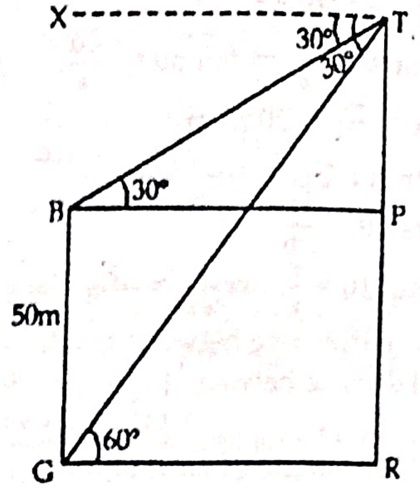

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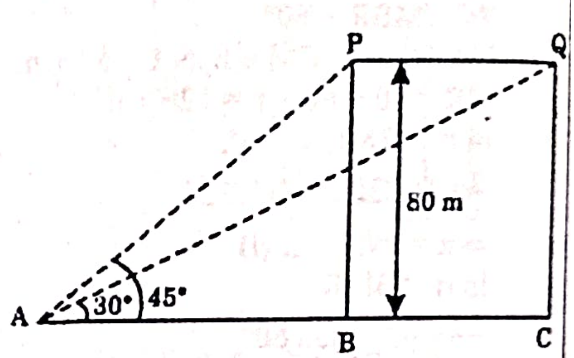

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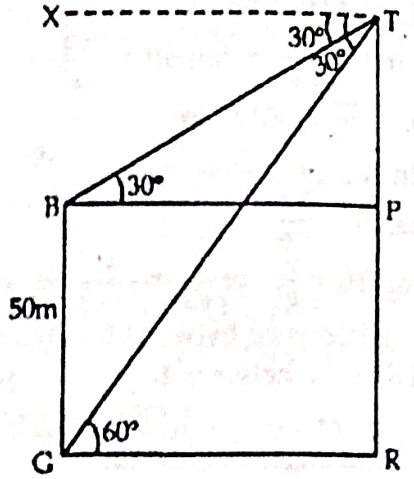

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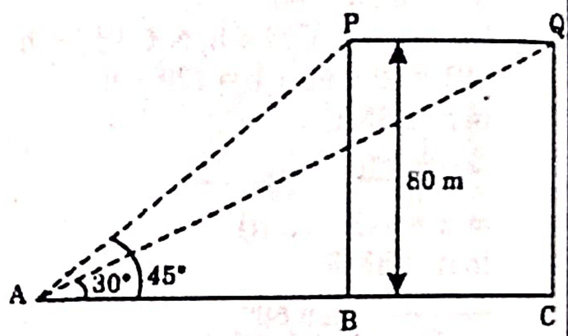

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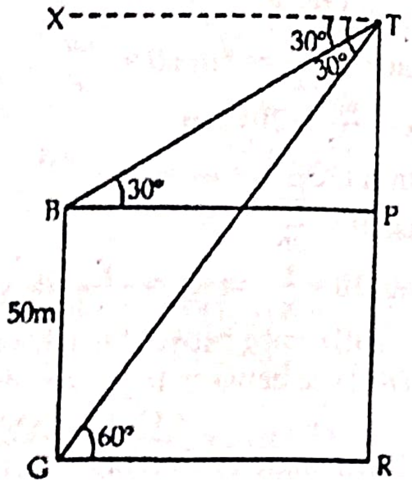

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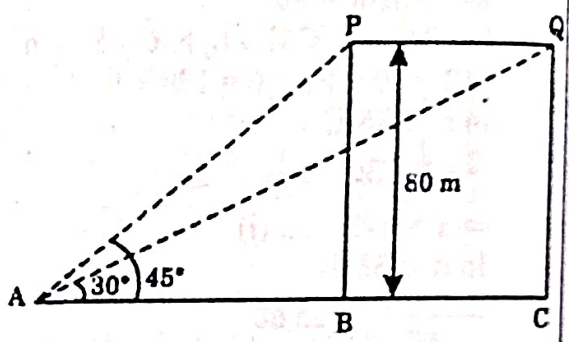

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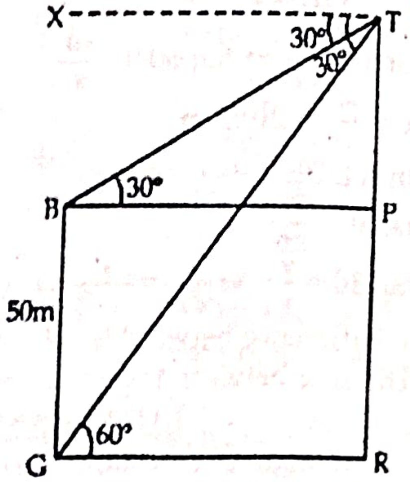

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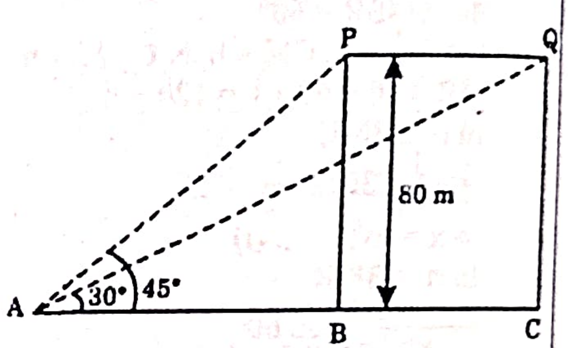

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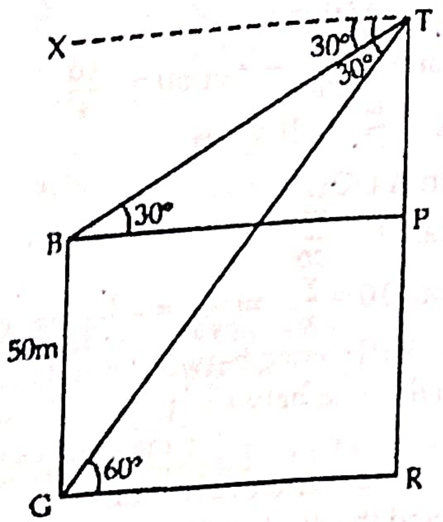

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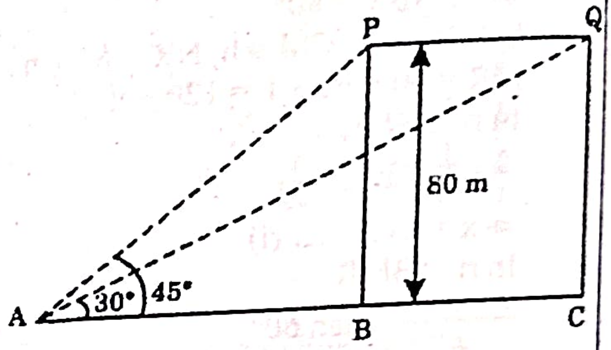

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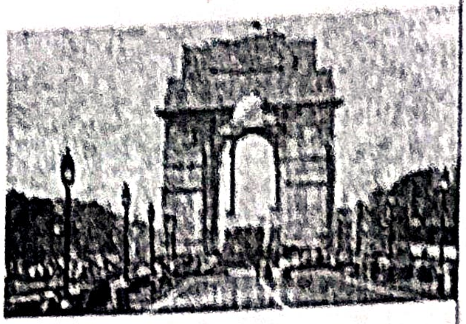
CASE BASED QUESTIONS (4 MARKS QUESTIONS)

1. A group of students of class X visited India Gate on an education trip. The teacher and students had interest in history as well. The teacher narrated that India Gate, official name Delhi Memorial, originally called All-India War Memorial, monumental sandstone arch in New Delhi, dedicated to the troops of British India who died in wars fought between 1914 and 1919. The teacher also said that India Gate, which is located at the eastern end of the Rajpath (formerly called the Kingsway), is about 138 feet (42 metres) in height

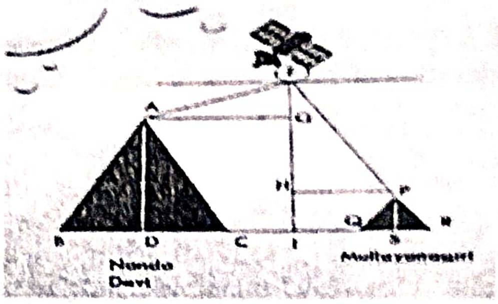


(i) What is the angle of elevation if they are standing at a distance of 42m away from the monument?
Solution: 45°
(ii)Find the length of the shadow cast by the India Gate if the elevation of the Sun is at 60°?
Solution: $14\sqrt{3}$ m
(iii)(a) The ratio of the length of the India Gate and its shadow is 1:1. The angle of elevation of the Sun is
Solution: 45°

(OR)

(iii)(b)Find the length of a ladder placed 42m away from the base of the India Gate such that its top just touches the tip of the monument ?
Solution: ~60m

2. A Satellite flying at height h is watching the top of the two tallest mountains in Uttarakhand and Karnataka, them being Nanda Devi(height 7,816m) and Mullayanagiri (height 1,930 m). The angles of depression from the satellite, to the top of Nanda Devi and Mullayanagiri are 30° and 60° respectively. If the distance between the peaks of the two mountains is 1937 km, and the satellite is vertically above the midpoint of the distance between the two mountains.



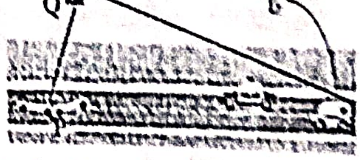
(i)Find the distance of the satellite from the top of Nanda Devi?
Solution: 1139.4 km
(ii) Find the distance of the satellite from the top of Mullayanagiri?
Solution:1937 km
(iii)(a) What is the angle of elevation if a man is standing at a distance of 7816m from Nanda Devi?
Solution: 45°

(OR)

(iii)(b)If a mile stone very far away from, makes 45° to the top of Mullanyangiri mountain. So, find the distance of this mile stone from the mountain.
Solution: 1937 km

3.A drone was used to facilitate movement of an ambulance on the straight highway to a point P on the ground where there was an accident. The ambulance was travelling at the speed of 60 km/h. The drone stopped at a point Q, 100 m vertically above the point P. The angle of depression of the ambulance was found to be 30° at a particular instant.

Based on above information, answer the following questions :



(i) Represent the above situation with the help of a diagram.
(ii) Find the distance between the ambulance and the site of accident (P) at the particular instant, (Use $\sqrt{3}$= 1.73)
Solution: Solution: The distance between the ambulance and the accident site is 173m
(iii)(a) Find the time (in seconds) in which the angle of depression changes from 30° to 45°.
Solution: The time in which the angle of depression changes from 30° to 45° is 4.38

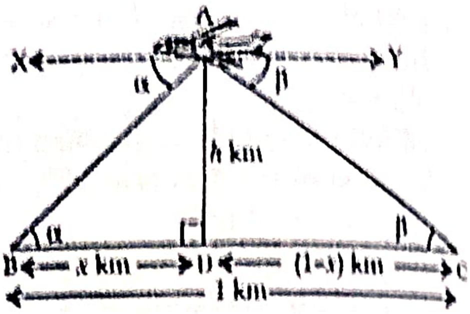
(OR)

(iii)(b)How long (in seconds) will the ambulance take to reach point P from a point T on the highway such that angle of depression of the ambulance at T is 60° from the drone?

Solution: The time it takes for the ambulance to reach point P from point T is 6.91 sec

HOTS (HIGH ORDER THINKING SKILLS)

1.From an aeroplane vertically above a straight horizontal plane, the angles of depression of two consecutive kilometres stones on the opposite sides of the aeroplane are found to be α and β. Show that the height of the aeroplane is $\frac{\tan\alpha\tan\beta}{\tan\alpha+\tan\beta}$



Solution: Let A be the aeroplane and its height be h km further, let B and C be two consecutive kilometres stone so that distance BC = 1 km.

Let BD = km

Then DC = (1 – x) km

In Δ ABD,

$\tan\alpha = \frac{AD}{BD} = \frac{h}{x} \Rightarrow x = \frac{h}{\tan\alpha}$ ...(i)

In Δ ADC,

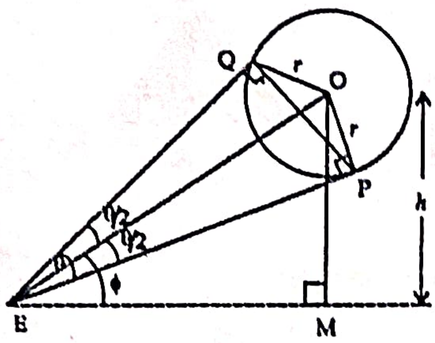
$\tan\beta = \frac{AD}{DC} = \frac{h}{1-x}$ ....(ii)

$\Rightarrow h = \tan\beta\,(1 - \frac{h}{\tan\alpha})$

From (i) and (ii)

$h = \tan\alpha\tan\beta - \frac{h\tan\beta}{\tan\alpha} \Rightarrow h(\tan\alpha+\tan\beta) = \tan\alpha\tan\beta \Rightarrow h = \frac{\tan\alpha\tan\beta}{\tan\alpha+\tan\beta}$

2.A spherical balloon of radius r subtends an angle 9 at the eye of an observer. If the angle of elevation of its centre is S, find the height of the centre of the balloon.



Solution: Let O be the centre of the balloon of radius . OP = r. Let E be the eye of an observer so that angle subtended by balloon at the eye E is ∠PEQ = 0 and angle of elevation of centre of ballon = ∠OEM = Φ

Let the height of the centre of balloon be h i.e, OM = h,

Let OE = d

$\therefore \angle PEO = \angle QEO = \frac{\theta}{2}$

Now, as radius is perpendicular to the tangent at point of contact .

∴ ∠OPE = 90°

So, ΔOPE is right angled at P and ΔOME is right angled at M.

In right ΔOPE, $\sin\left(\frac{\theta}{2}\right) = \frac{OP}{OE} = \frac{r}{d}$

In Δ OME,

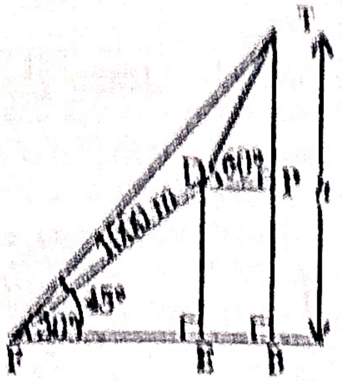
$\sin\Phi = \frac{OM}{OE} = \frac{h}{d}$

$\frac{\sin\sin(\Phi)}{\sin\sin(\frac{\theta}{2})} = \frac{h}{r}$

$\Rightarrow h = r \sin\phi \operatorname{cosec}\left(\frac{\theta}{2}\right)$

Hence, the height of centre of the balloon is $r \sin\phi \operatorname{cosec}\left(\frac{\theta}{2}\right)$.

3. At the foot of a mountain the elevation of its summit is 45°, after ascending 1000 m towards the mountain up a slope of 30° inclination, the elevation is found to be 60°. Find the height of the mountain.



Solution: Let F be the foot and T be the summit of the mountain TFH such that $\angle TFH = 45^\circ$

$\therefore$ In right $\triangle TBF$, $\angle BTF = 90^\circ - 45^\circ = 45^\circ$

Let height of mountain be h i.e., TB = h

since $\angle TFH = \angle BTF = 45^\circ \Rightarrow BF = BT = h$

$\angle DFE = 30^\circ$ and $FD = 1000$ m, $\angle TDP = 60^\circ$

Draw $DE \perp BF$ and $DP \perp BT$

In right $\triangle DEF$, $\cos 30^\circ = \frac{FE}{DF} \Rightarrow \frac{\sqrt{3}}{2} = \frac{FE}{1000} \Rightarrow FE = 500\sqrt{3}$ m

Also $\sin 30^\circ = \frac{DE}{DF} \Rightarrow \frac{1}{2} = \frac{DE}{1000} \Rightarrow DE = 500$ m

$\therefore BP = DE \Rightarrow BP = 500$ m

Now $TP = TB - BP = h - 500$

In $\triangle TPD$, $\cos 60^\circ = \frac{DP}{TP} \Rightarrow DP = \frac{h-500}{\sqrt{3}}$

Now, $BF = BE + EF = DP + EF$

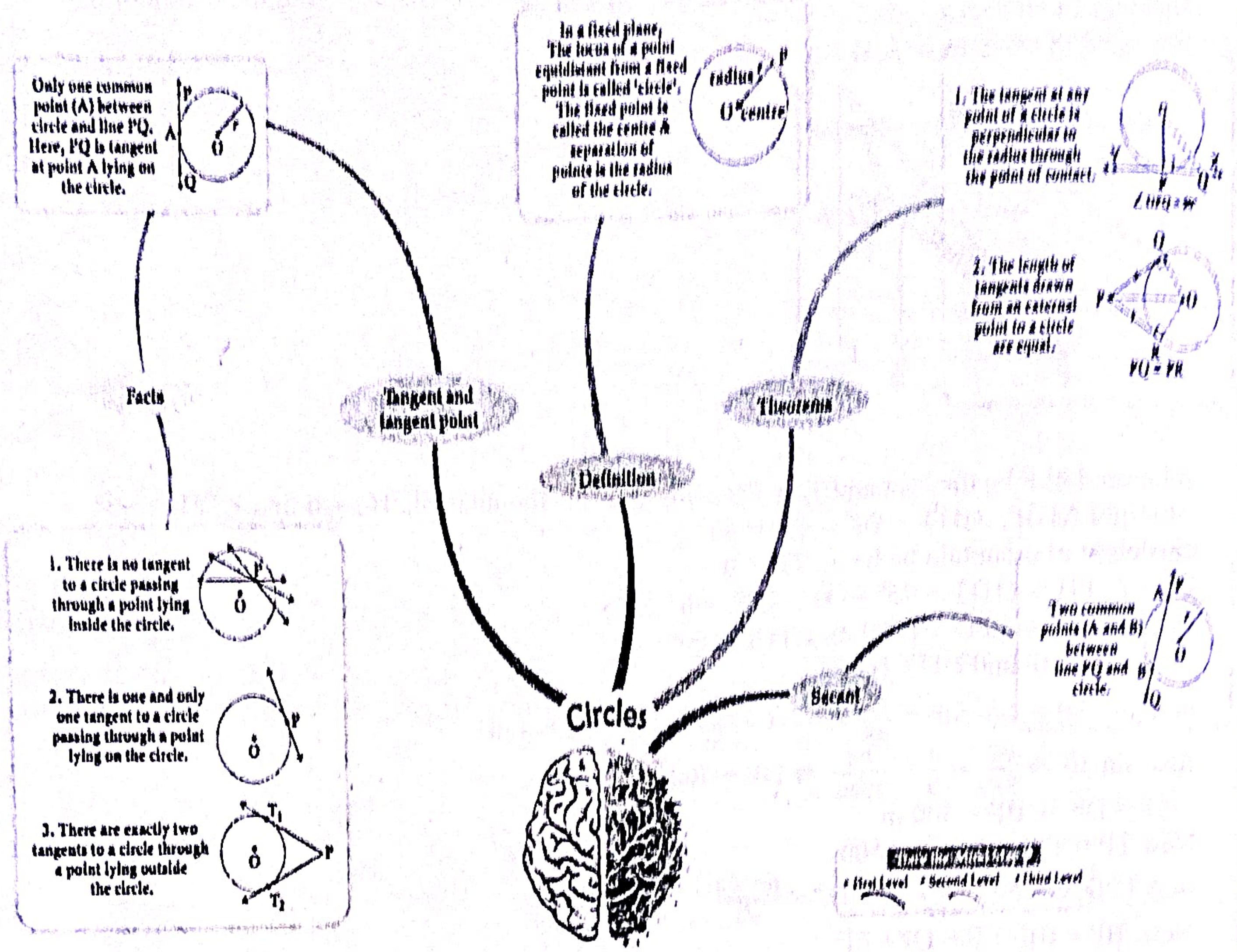
In $\triangle TBF$, $\tan 45 = \frac{TB}{BF} \Rightarrow TB = BF = DP + EF$

$h = \frac{h-500}{\sqrt{3}} + 500\sqrt{3} = \frac{h-500+1500}{\sqrt{3}} = \frac{h+1000}{\sqrt{3}}$

$\sqrt{3}\,h - h = 1000 = h(1.732 - 1) = 1000$

$h = \frac{1000}{0.732} = 1366$ m

CHAPTER – 10
CIRCLES
MIND MAP



GIST OF THE CHAPTER :-

(1)Introduction
(2)Tangents to a circle
(3)The tangent at any point of a circle is perpendicular to the radius through the point of contact.
(4)Number of Tangents from a Point on a Circle
(5)The lengths of tangents drawn from an external point to a circle are equal.

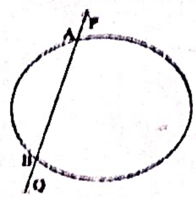
DEFINITIONS :-

(1) **Circle** :- A circle is a collection of all points in a plane which are at a constant distance (radius) from a fixed point(centre)

(2) **Chord :-** The chord is the line segment having its two end points lying on the circumference of the circle.

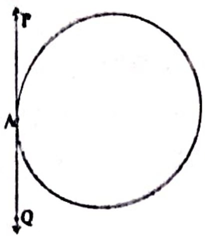
(3) **Secant** :- A secant to a circle is a line that intersects the circle at exactly two points.

A line PQ is called a secant.



NOTE :- A secant to a circle is a line where as a chord of a circle is a line segment whose end

(4) **Tangent to a circle** :- A tangent to a circle is a line that intersects the circle at only one point.



A line PQ is called a tangent to a circle.

(5) **Point of Contact** :- The common point of the tangent and the circle is called the point of contact. In the above figure the point A is called the point of contact.

NOTE :- (i) There is only one tangent at a point of the circle. There are infinitely many points on a circle, so a circle can have infinitely many tangents.

(ii) The tangent to a circle is a special case of the secant, when the two end points of its corresponding chord coincides.

(iii) A circle can have two (2) parallel tangents at the most.

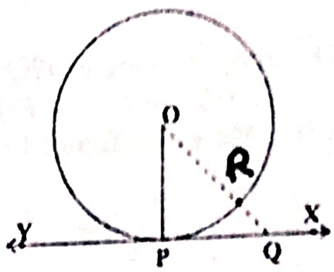
(6) **Theorems** :-

Theorem 10.1 :- The tangent at any point of a circle is perpendicular to the radius through the point of contact.

Given :- A circle with centre O and a tangent XY to the circle at a point P.

To Prove :- $OP \perp XY$

Construction :- Take a point Q on XY other than P and join OQ. Suppose OQ meets the circle at R



Proof :- Among all line segments joining the point O to a point on XY, the shortest one is perpendicular to XY. So, to prove that OP ⊥ XY, it is sufficient to prove that OP is shorter than any other segment joining O to any point of XY.

Clearly OP = OR.[radii of the same circle]

Now, OQ = OR + RQ

⇒ OQ > OR

⇒ OQ > OP (∴ OP = OR)

Thus, OP is shorter than any other segment joining O to any point of XY.

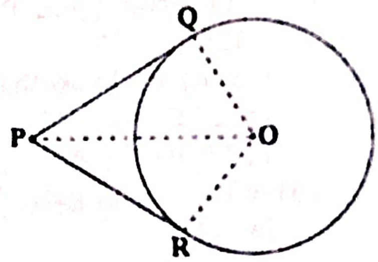
Hence, OP ⊥ XY.

Theorem 10.2 :- The lengths of tangents drawn from an external point to a circle are equal.

Given :- A circle with centre O, a point P lying outside the circle and two tangents PQ, PR on the circle from P

To Prove :- PQ = PR

Construction :- Join OP , OQ and OR



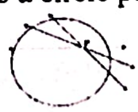
Proof :- In Δ PQO and Δ PRO

$\angle PQO = \angle PRO = 90^0$ [radius is perpendicular to the tangent at the point of contact]

PO = PO	[Common side]
OQ = OR	[radii of the same circle]
Δ PQO ≅ Δ PRO	[by RHS congruence rule]
PQ = PR	[CPCT]

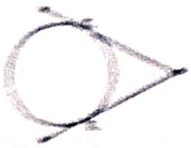
Number of tangents from a point on a circle :-

(i)There is no tangent to a circle passing through a point lying inside the circle.



(ii)There is one and only one tangent to a circle passing through a point lying on the circle.

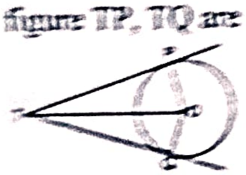
(ii) There are exactly two tangents to a circle through a point lying outside the circle.



(7) **Length of the tangent** : The length of the segment of the tangent from the external point P and the point of contact with the circle is called the length of the tangent from the point P to the circle.

(8) In right triangle OPQ, $\angle OPQ = 90^0$, *OQ is the hypotenuse then*
(i) $OQ^2 = OP^2 + PQ^2$ (ii) $OP^2 = OQ^2 - PQ^2$ (iii) $PQ^2 = OQ^2 - OP^2$

(9) In the given figure TP, TQ are two tangents drawn from an external point T.

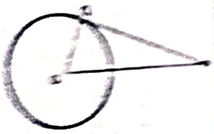


(i) TO bisects angle PTQ and angle POQ (ii) $\angle PTQ + \angle POQ = 180^0$

MULTIPLE CHOICE QUESTIONS(1 MARK QUESTIONS)

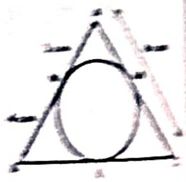
(1) Two parallel lines touch the circle at points A and B respectively. If area of the circle is 25π cm^2, then AB is equal to
(a) 5 cm (b) 8 cm (c) 10 cm (d) 25 cm
Ans) (c) Area of a circle $= \pi r^2 = 25\pi \Rightarrow r = 5$ cm, Distance AB = 10cm

(2) In the given figure PQ is tangent then $\angle POQ + \angle QPO$ is



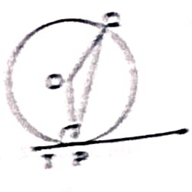
(a) 180° (b) 45° (c) more than 90° (d) 90°
Ans) (d) We know that a radius is perpendicular to the tangent at the point of contact,
so $\angle PQO = 90^0$
In ΔPQO, by ASP *of a triangle*, $\angle POQ + \angle QPO = 90^0$

(3) In the given figure, ABC is circumscribing a circle, then the length of BC is



(a) 4 cm (b) 8 cm (c) 9 cm (d) 5 cm
Ans) (c)
The lengths of the tangents drawn from an external point to a circle are equal.
BN = BL = 4 cm , AN = AM = 3 cm , CL = CM = 8 - 3 = 5 cm
BC = BL + CL = 4 + 5 = 9 cm

(4) In the given figure, a tangent has been drawn at a point P on the circle centred at O.
If ∠ TPQ = 110° then ∠POQ is equal to



(a) 110° (b) 70° (c) 140° (d) 55°
Ans) (c) We know that $OP \perp TP$, $\angle OPQ = 110^0 - 90^0 = 20^0$
Since, OP = OQ, so $\angle OPQ = \angle OQP = 20^0$
By ASP of a triangle $\angle POQ = 180^0 - (20 + 20) = 180^0 - 40^0 = 140^0$

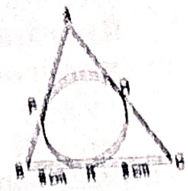
(5)In the given figure, PQ and PR are tangents to a circle centred at O. If $\angle QPR = 35^0$ then $\angle QOP$ is equal to

(a) 72.5^0 (b) 73.5^0 (c) 135^0 (d) 145^0

Ans)(a) $\angle QPR + \angle QOR = 180^0$, $\angle QOR = 180^0 - 35^0 = 145^0$

PO bisects $\angle QOR$ so , $\angle QOP = \frac{1}{2} \angle QOR = 72.5^0$

(6)If perimeter of given triangle is 38 cm, then length AP is equal to

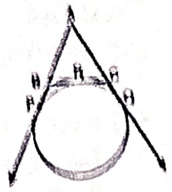


(a)19 cm (b) 5 cm (c) 10 cm (d) 8 cm

Ans) Let AP = AQ = x

2x + 12 + 16 = 38 cm 2x = 38 – 28 = 10 cm $x = \frac{10}{2} = 5$ cm

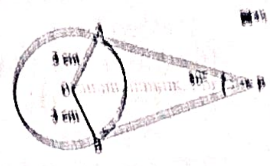
(7)In figure, AP, AQ and BC are tangents to the circle. If AB = 5 cm, AC = 6 cm and BC = 4 cm, then the length of AP (in cm) is



(a) 15 (b) 7.5 (c) 20 (d) 30

Ans)(b) AP + AQ = Perimeter of ΔABC, AP = ½ (Perimeter of ΔABC) = 7.5 cm

(8) If two tangents inclined at an angle 60° are drawn to a circle of radius 3 cm, then length of each tangent is equal to

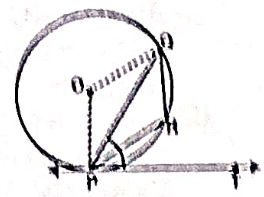


(a) (3/2)√3 cm (b) 6 cm (c) 3 cm (d) 3√3 cm

Ans)(d) OP bisects $\angle APB \Rightarrow \angle APO = \frac{60}{2} = 30^0$, In rt ΔOAP, $\tan 30^0 = \frac{OA}{AP}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{3}{AP} \Rightarrow AP = 3\sqrt{3}$ cm

(9)In the figure below, PQ is a chord of a circle and PT is the tangent at P such that $\angle QPT = 60°$, Then $\angle PRQ$ is equal to



(a) 135° (b) 150° (c) 120° (d) 110°

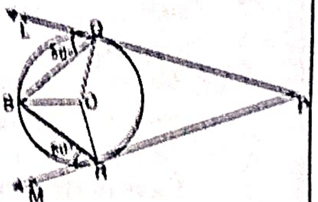
Ans) (c)From the given, $\angle QPT = 60°$ and $\angle OPT = 90°$

Thus, $\angle OPQ = \angle OQP = 30°$, i.e., $\angle POQ = 120°$.

Also, $\angle PRQ = \frac{1}{2} \times$ reflex $\angle POQ$, reflex $\angle POQ = 360° - 120° = 240°$

Therefore, $\angle PRQ = \frac{1}{2} \times 240° = 120°$

(10)In the figure, PQL and PRM are tangents to the circle with centre O at the points Q and R, respectively and S is a point on the circle such that $\angle SQL = 50°$ and $\angle SRM = 60°$. Then $\angle QSR$ is equal to



(a) 40° (b) 60° (c) 70° (d) 80°

Ans)(c) From the given,

$\angle OSQ = \angle OQS = 90° - 50° = 40°$

and $\angle RSO = \angle SRO = 90° - 60° = 30°$

Therefore, $\angle QSR = 40° + 30° = 70°$.

ASSERTION REASON BASED QUESTION

DIRECTION: In the following questions, a statement of **Assertion (A)** is followed by a statement of **Reason (R).** Choose the correct option

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

(1) **Assertion(A):** If length of a tangent from an external point to a circle is 8 cm, then length of the other tangent from the same point is 8 cm.

Reason(R): length of the tangents drawn from an external point to a circle are equal.

Ans) (a)

(2) **Assertion(A):** The length of the tangent drawn from a point at a distance of 13 cm from the centre of a circle of radius 5 cm is 10 cm.

Reason(R): A tangent to a circle is perpendicular to the radius through the point of contact.

Ans) (d)length of the tangent = 12 cm

(3) **Assertion(A):** PA and PB are tangents drawn from an external point P to a circle with centre O such that $\angle APB = 40^\circ$, then $\angle AOB = 170^\circ$.

Reason(R): The angle between the two tangents drawn from an external point at a circle are supplementary to the angle subtended by the line segments joining the points of contact at the centre.

Ans) (d)

(4) **Assertion(A):** If the radius of a circle is 5 cm then the distance between parallel tangents is 10 cm

Reason (R) : The radius of a circle is perpendicular to the tangent at the point of contact.

Ans) (b)

(5) **Assertion (A):** If two tangents are drawn to a circle from an external point, then the angles between the tangents and the chord joining the points of contact are equal.

Reason (R): The tangents drawn from an external point to a circle are always equal in length.

Ans) (a)

(6) **Assertion (A):** The tangents drawn at the ends of a diameter of a circle are parallel.

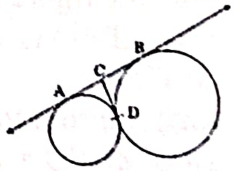
Reason (R): The angle between the tangent and the radius at the point of contact is 90°

Ans) (a)

(7) **Assertion (A):** In the below figure, AB and CD are common tangents to circles which touch each other at D. If AB = 8 cm, then the length of CD is 4 cm.

Reason (R): The tangents drawn from an external point to a circle are always equal in length.

Ans) (a)



(8) **Assertion(A):** If PA and PB are tangents drawn from an external point P to a circle with centre O, then the quadrilateral AOBP is cyclic.

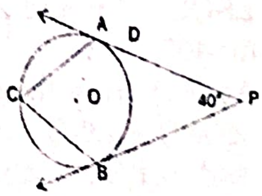
Reason(R): The angle between the two tangents drawn from an external point at a circle are supplementary to the angle subtended by the line segments joining the points of contact at the centre.

Ans) (a)

(9)**Assertion(A):** In figure, PA and PB are tangents drawn from an external point P to a circle with centre O such that $\angle APB = 40^\circ$. If C is a point on the circle, then $\angle ACB = 70^\circ$.

Reason (R): The angle subtended by an arc at the centre is double to the angle subtended by it at any point on the remaining part of the circle

Ans) (a)



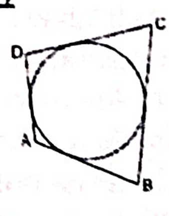
(10)**Assertion:** A tangent to a circle is perpendicular to the radius through the point of contact.

Reason: The lengths of tangents drawn form an external point to a circle is equal.

Ans)(b)

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS QUESTIONS)

(1) In figure, a circle touches all the four sides of a quadrilateral ABCD whose sides are AB = 6 cm, BC = 9 cm and CD = 8 cm. Find the length of side AD.



Ans) $AD + BC = AB + DC \Rightarrow AD = 5$ cm

(2) **Assertion(A):** The length of the tangent drawn from a point at a distance of 13 cm from the centre of a circle of radius 5 cm is 10 cm.

Reason(R): A tangent to a circle is perpendicular to the radius through the point of contact.

Ans) (d)length of the tangent = 12 cm

(3) **Assertion(A):** PA and PB are tangents drawn from an external point P to a circle with centre O such that ∠APB = 40°, then ∠AOB = 170°.

Reason(R): The angle between the two tangents drawn from an external point at a circle are supplementary to the angle subtended by the line segments joining the points of contact at the centre.

Ans) (d)

(4) **Assertion(A):** If the radius of a circle is 5 cm then the distance between parallel tangents is 10 cm

Reason (R) : The radius of a circle is perpendicular to the tangent at the point of contact.

Ans) (b)

(5) **Assertion (A):** If two tangents are drawn to a circle from an external point, then the angles between the tangents and the chord joining the points of contact are equal.

Reason (R): The tangents drawn from an external point to a circle are always equal in length.

Ans) (a)

(6) **Assertion (A):** The tangents drawn at the ends of a diameter of a circle are parallel.

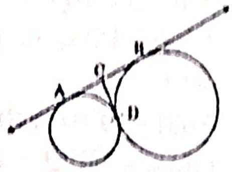
Reason (R): The angle between the tangent and the radius at the point of contact is 90°

Ans) (a)

(7) **Assertion (A):** In the below figure, AB and CD are common tangents to circles which touch each other at D. If AB = 8 cm, then the length of CD is 4 cm.

Reason (R): The tangents drawn from an external point to a circle are always equal in length.

Ans) (a)



(8) **Assertion(A):** If PA and PB are tangents drawn from an external point P to a circle with centre O, then the quadrilateral AOBP is cyclic.

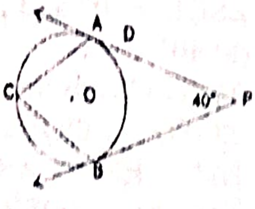
Reason(R): The angle between the two tangents drawn from an external point at a circle are supplementary to the angle subtended by the line segments joining the points of contact at the centre.

Ans) (a)

(9)**Assertion(A):** In figure, PA and PB are tangents drawn from an external point P to a circle with centre O such that ∠APB = 40°. If C is a point on the circle, then ∠ACB = 70°.

Reason (R): The angle subtended by an arc at the centre is double to the angle subtended by it at any point on the remaining part of the circle

Ans) (a)



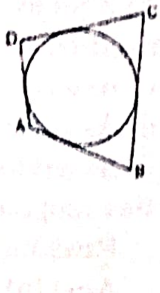
(10)**Assertion:** A tangent to a circle is perpendicular to the radius through the point of contact.

Reason: The lengths of tangents drawn form an external point to a circle is equal.

Ans)(b)

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS QUESTIONS)

(1) In figure, a circle touches all the four sides of a quadrilateral ABCD whose sides are AB = 6 cm, BC = 9 cm and CD = 8 cm. Find the length of side AD.



Ans) AD + BC = AB + DC ⟹ AD = 5 cm

(2) **Assertion(A):** The length of the tangent drawn from a point at a distance of 13 cm from the centre of a circle of radius 5 cm is 10 cm.
Reason(R): A tangent to a circle is perpendicular to the radius through the point of contact.
Ans) (d)length of the tangent = 12 cm

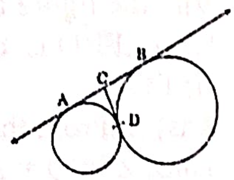
(3) **Assertion(A):** PA and PB are tangents drawn from an external point P to a circle with centre O such that $\angle APB = 40°$, then $\angle AOB = 170°$.
Reason(R): The angle between the two tangents drawn from an external point at a circle are supplementary to the angle subtended by the line segments joining the points of contact at the centre.
Ans) (d)

(4) **Assertion(A):** If the radius of a circle is 5 cm then the distance between parallel tangents is 10 cm
Reason (R) : The radius of a circle is perpendicular to the tangent at the point of contact.
Ans) (b)

(5) **Assertion (A):** If two tangents are drawn to a circle from an external point, then the angles between the tangents and the chord joining the points of contact are equal.
Reason (R): The tangents drawn from an external point to a circle are always equal in length.
Ans) (a)

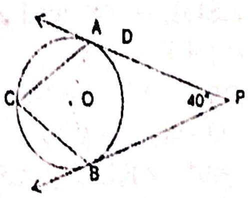
(6) **Assertion (A):** The tangents drawn at the ends of a diameter of a circle are parallel.
Reason (R): The angle between the tangent and the radius at the point of contact is 90°
Ans) (a)

(7) **Assertion (A):** In the below figure, AB and CD are common tangents to circles which touch each other at D. If AB = 8 cm, then the length of CD is 4 cm.
Reason (R): The tangents drawn from an external point to a circle are always equal in length.
Ans) (a)



(8) **Assertion(A):** If PA and PB are tangents drawn from an external point P to a circle with centre O, then the quadrilateral AOBP is cyclic.
Reason(R): The angle between the two tangents drawn from an external point at a circle are supplementary to the angle subtended by the line segments joining the points of contact at the centre.
Ans) (a)

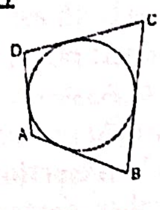
(9)**Assertion(A):** In figure, PA and PB are tangents drawn from an external point P to a circle with centre O such that $\angle APB = 40°$. If C is a point on the circle, then $\angle ACB = 70°$.
Reason (R): The angle subtended by an arc at the centre is double to the angle subtended by it at any point on the remaining part of the circle
Ans) (a)



(10)**Assertion:** A tangent to a circle is perpendicular to the radius through the point of contact.
Reason: The lengths of tangents drawn form an external point to a circle is equal.
Ans)(b)

<u>VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS QUESTIONS)</u>

(1) In figure, a circle touches all the four sides of a quadrilateral ABCD whose sides are AB = 6 cm, BC = 9 cm and CD = 8 cm. Find the length of side AD.

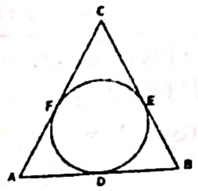


Ans) $AD + BC = AB + DC \Rightarrow AD = 5$ cm

(2) In the given figure, if AB= AC, prove that BE = EC.

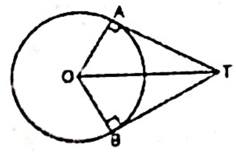
Ans) Given that AB = AC $\Longrightarrow$ AD + DB = AF + FC $\Rightarrow$ AD + DB = AD + FC (tangents from an external point are equal)

DB = FC $\Rightarrow$ BE = EC (tangents from an external point are equal)



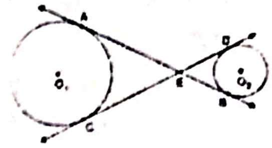
(3) In figure if $\angle ATO = 40°$, find $\angle AOB$.

Ans) $\angle ATB = 2 X 40 = 80^0$, $\angle AOB = 180^0 - 80^0 = 100^0$



(4)In figure, common tangents AB and CD to the two circle O_1 and O_2 intersect at E. Prove that AB = CD.

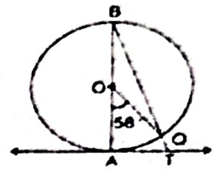
Ans) EA = EC and EB = ED $\Rightarrow EA + EB = EC + ED \Rightarrow AB = CD$



(5) In given figure, AB is the diameter of a circle with centre O and AT is a tangent. If $\angle AOQ = 58°$, find $\angle ATB$.

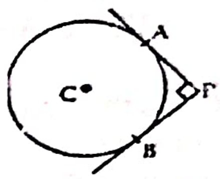
Ans) $\angle ABQ = \frac{1}{2} X 58 = 29^0$

$\angle BAT = 90^0$, $\angle ATB = 180^0 - (90 + 29) = 61^0$



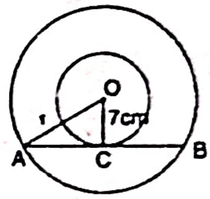
(6) In the given figure, PA and PB are two tangents drawn from an external point P to a circle with centre C and radius 4 cm. If PA ⊥ PB, then find the length of each tangent.

Ans) Join CA and CB , then CAPB is a square. PA = PB = 4 cm.



(7) Two concentric circles are of radii 7 cm and r cm respectively, where r > 7 cm . A chord of the larger circle, of length 48 cm, touches the smaller circle.
Find the value of r.

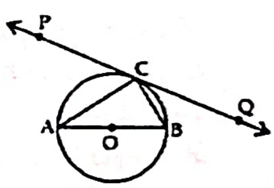
Ans)AC = BC = 24 cm , By using Pythagoras theorem , OA = r = 25 cm



(8) In the given figure, PQ is a tangent at a point C to a circle with centre O. If AB is a diameter and $\angle CAB = 30°$. Find $\angle PCA$.

Ans) Join OC , OA = OC $\Longrightarrow \angle OAC = \angle OCA = 30^0$

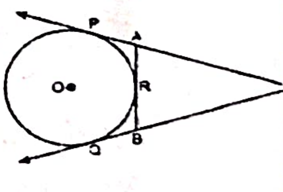
$\angle PCA = 90^0 - 30^0 = 60^0$



(9) In figure, CP and CQ are tangents to a circle with centre O. ARB is another tangent touching the circle at R. If CP = 11 cm, and BC = 7 cm, then find the length of BR.

Ans) CQ = CP = 11 cm $\Longrightarrow$ CB + BQ = 11 cm $\Longrightarrow$ CB + BR = 11 cm $\Longrightarrow$

BR = 11 cm – 7 cm = 4 cm.

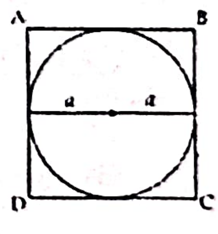


(10) Find the perimeter (in cm) of a square circumscribing a circle of radius 'a' cm.

Ans) Radius = R

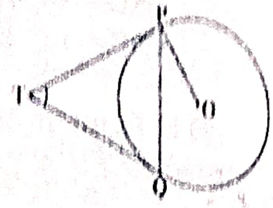
AB = a + a = 2a

∴ Perimeter = 4(AB) = 4(2a) = 8a cm



SHORT ANSWER TYPE OF QUESTIONS (3 MARKS QUESTIONS)

(1) Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.



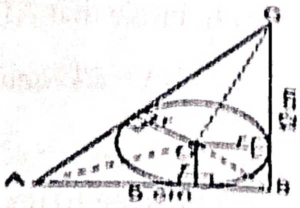
Ans) Let $\angle PTQ = x$, Now, TP = TQ. So TPQ is an isosceles triangle.

Therefore, $\angle TPQ = \angle TQP = ½ (180^\circ - x) = 90^\circ - ½ x$ and $\angle OPT = 90^\circ$

$\angle OPQ = \angle OPT - \angle TPQ = 90^\circ - (90^\circ - ½ x) = ½ x = ½ \angle PTQ$,

$\angle PTQ = 2\angle OPQ$

(2) In a right triangle ABC, right-angled at B, BC = 12 cm and AB = 5 cm. Calculate the radius of the circle inscribed in the triangle (in cm).



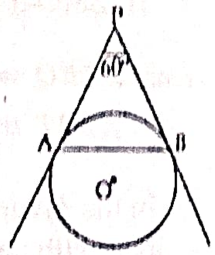
Ans) AC = 13 cm , (By Pythagoras theorem)

Area of ΔABC = Area of ΔAOB + ar. of ΔBOC + ar. of ΔAOC

60 = r (AB + BC + AC)

60 = 30r ⇒ r = 2 cm

(3) In the figure, AP and BP are tangents to a circle with centre O, such that AP = 5 cm and $\angle APB = 60^\circ$. Find the length of chord AB.



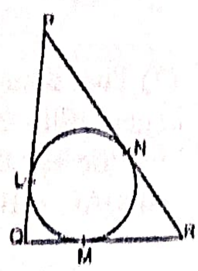
Ans) PA = PB ...[Tangents drawn from an external point are equal]

⇒ $\angle PAB = \angle PBA = \angle APB = 60^\circ$

∴ ΔPAB is an equilateral triangle

Hence, AB = AP = 5 cm ...[∵ All sides of an equilateral triangle are equal]

(4) In the figure, a circle is inscribed in a triangle PQR with PQ = 10 cm, QR = 8 cm and PR = 12 cm. Find the lengths of QM, RN and PL.



Ans) Let PL = PN = x cm , QL = QM = y cm , RN = MR = z cm

PQ = 10 cm = x + y = 10 ...(i) , QR = 8 cm = y + z = 8 ...(ii)

PR = 12 cm = x + z = 12 ...(iii)

By adding (i), (ii) and (iii),

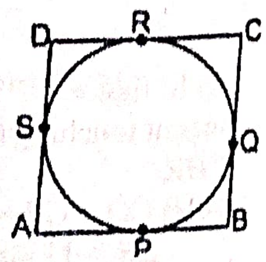
2x + 2y + 2z = 10 + 8 + 12 ⇒ 2(x + y + z) = 30

⇒ x + y + z = 15 ⇒ 10 + z = 15 ... [From (i)]

∴ z = 15 − 10 = 5 cm, y = 3 cm, x = 7 cm

QM = 3cm, RN = 5cm and PL = 7cm

(5) A corporation of Amaravathi city has allocated a parallelogram ABCD shaped land to construct a circular cricket stadium touching all the sides of ABCD to BCCI. Show that ABCD is a rhombus and find its area if AC = 2.5 km and BD = 3.2 km.



Ans) ABCD is a ‖gm

AP = AS , BP = BQ , CR = CQ , DR = DS [By known theorem]

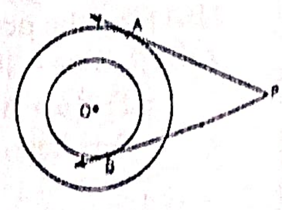
By adding the above we get AB + DC = AD + BC ⇒ $2\,AB = 2\,BC$

$\Rightarrow AB = BC$

Therefore , ABCD is a rhombus.

Area of rhombus = $\frac{1}{2}$ $X\ product\ of\ diagonals = \frac{1}{2}\ X\ 2.5\ X\ 3.2 = 4\ sq\ km$.

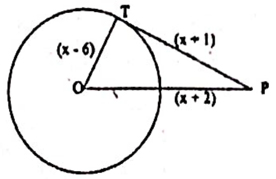
(6) In figure, there are two concentric circles, with centre O and of radii 5 cm and 3 cm. From an external point P, tangents PA and PB are drawn to these circles. If AP = 12 cm, find the length of BP.



Ans) Join OA , OB and OP

By Pythagoras theorem , OP = 13 cm . BP = $\sqrt{169 - 9} = \sqrt{160} = 4\sqrt{10}$ cm.

(7) In the below figure, find the actual length of sides of ⊗OTP

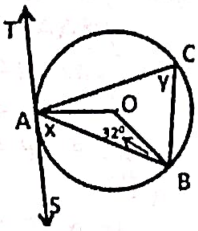


Ans) By using Pythagoras theorem , $OP^2 = OT^2 + TP^2$ we get $x^2 - 14x + 33 = 0$

By solving the equation we get x = 11 and x = 3 (rejected)

OP = 13 cm , OT = 5 cm , TP = 12 cm

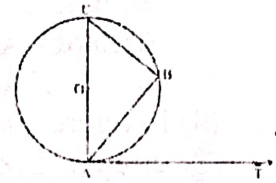
(8) In the given figure TAS is a tangent to the circle, with centre O, at the point A. If ∠OBA = 32°, find the value of x and y.



Ans) $\angle OBA = \angle OAB = 32^0$; $x = 90^0 - 32^0 = 58^0$

$y = \frac{1}{2}$ X $\angle AOB = 58^0$

(9)If AB is a chord of a circle with centre O, AOC is a diameter and AT is the tangent at A as shown in Fig. Prove that ∠BAT = ∠ACB



Ans) $\angle ABC = 90^0 (angle\ in\ a\ semi-circle)$

$\angle ACB = 90^0 - \angle BAC$ ....(1) $\angle BAT = 90^0 - \angle BAC$ ...(2)

From (1) and (2) we get $\angle BAT = \angle ACB$

(10) At one end A of a diameter AB of a circle of radius 5 cm, tangent XAY is drawn to the circle. Find the length of the chord CD parallel to XY and at a distance 8 cm from A.

Ans) A chord CD is drawn which is parallel to XY and at a distance of 8 cm from A.

Now, ∠OAY = 90°

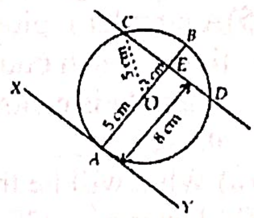
[∵ Tangent at any point of circle is perpendicular to the radius through the point of contact]

Now, in right angled ΔOEC,

$OC^2 = OE^2 + EC^2$ [By Pythagoras theorem] ⇒ $EC^2 = OC^2 - OE^2$

⇒ $EC^2 = 5^2 - 3^2$

⇒ $EC^2 = 25 - 9 = 16$



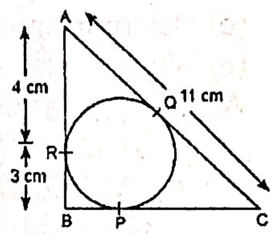
⇒ EC = 4 cm

CD = 2 x EC ⇒ CD = 2 x 4 ⇒ CD = 8 cm

LONG ANSWER TYPE QUESTIONS (5 MARKS QUESTIONS)

(1) (a) Prove that the lengths of tangents drawn from an external point to a circle are equal. (3M)

(b) In figure, ΔABC is circumscribing a circle. Find the length of BC. (2M)



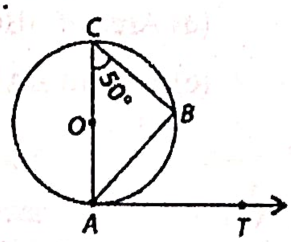
Ans) (a) proof already given in the gist of lesson

(b) BP = BR = 3 cm and CP = CQ = 11 – 4 = 7 cm

BC = 7 + 3 = 10 cm.

(2)(a) Prove that the radius of a circle is perpendicular to the tangent at the point of contact. (3M)

(b) In figure, AB is a chord of the circle and AOC is its diameter such that ∠ACB = 50°. If AT is the tangent to the circle at the point A, then find ∠BAT.



Ans) (a) proof already given in the gist of lesson

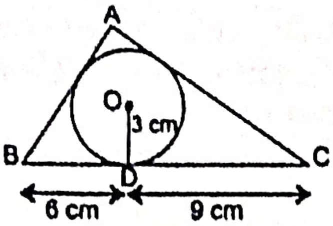
(b) ∠ABC = 90°

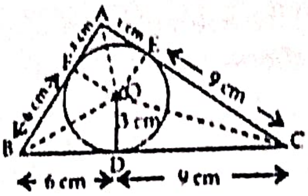
In ΔACB, ∠A + ∠B + ∠C = 180° [Angle sum property]

⇒ ∠A + 90° + 50° = 180° ⇒ ∠A + 140° = 180° ⇒ ∠OAB = 40°

∠BAT =90° - 40° = 50°

(3) In the figure, a ΔABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If the area of ΔABC is 54 cm^2, then find the lengths of sides AB and AC.





Ans) Given: OD = 3 cm; OE = 3 cm; OF = 3 cm

$ar(\Delta ABC) = 54\ cm^2$

Join : OA, OF, OE, OB and OC

Let AF = AE = x, BD = BF = 6 cm , CD = CE = 9 cm

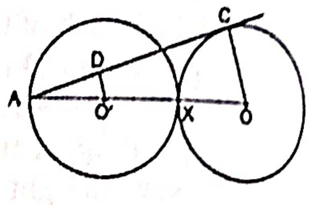
$\therefore AB = AF + BF = x + 6$ ...(i) $AC = AE + CE = x + 9$ ...(ii) $BC = DB + CD = 6 + 9 = 15$ cm ...(iii)

In ΔABC,

Area of $\Delta ABC = 54\ cm^2$ ...[Given] $ar(\Delta ABC) = ar(\Delta BOC) + ar(\Delta AOC) + ar(\Delta AOB)$

$x = 3$, AB = 9 cm and AC = 12 cm.

(4) In figure, two equal circles, with centres O and O′, touch each other at X. OO′ produced meets the circle with centre O′ at A. AC is tangent to the circle with centre O, at the point C. O′D is perpendicular to AC. Find the value of $\frac{O'D}{OC}$.

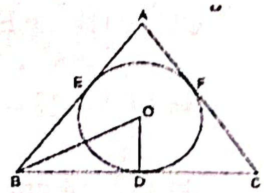


Ans) $\angle ADO' = \angle ACO = 90^0$ *and* $\angle DAO' = \angle CAO$ (common angle)

$\Delta ADO' \sim \Delta ACO$ $\frac{DO'}{CO} = \frac{AO'}{AO}$

$\frac{DO'}{CO} = \frac{r}{3r} = \frac{1}{3}$

(5)A circular region is inscribed in a triangular boundary as shown in figure. Each boundary of triangular part is act as tangent to the circle, where O is centre of circle and OD ⊥BC. Answer the questions based on above



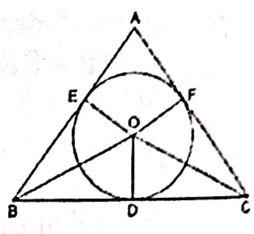
(a) What will be the radius of the circle, if BD = 24 cm and OB = 25 cm?
(b) Determine CD, if OC = 26 cm.
(c) As AB and AC act as tangents to the circle at E and F and AE = 8 cm, then what is the perimeter of ΔABC.
(d) Determine area of ΔBOC.
(e) What is the area of ΔABC?

Ans) (a) By Pythagoras theorem, OD = 7 cm.

(b) $CD^2 = 676 - 49 = 627\ cm^2 \Rightarrow CD = 25.04$ cm

(c) As BD = BE and CD = CF ⇒ BE = 24 cm

CF = 25.04 cm AE = AF = 8 cm



Perimeter of ΔABC = AB + BC + AC

= (AE + BE) + (BD + CD) + (AF + FC)

= (8 + 24) + (24 + 25.04) + (8 + 25.04) = 32 + 49.04 + 33.04 = 114.08 cm

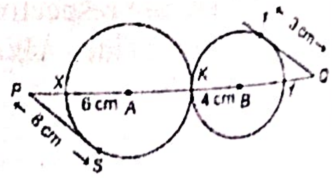
(d) Area of $\Delta BOC = \frac{1}{2} \times BC \times OD = \frac{1}{2}\ (49.04) \times 7 = 171.64\ cm^2$

(e) Area of $\Delta ABC = \frac{1}{2} \times$ (Perimeter of ΔABC) × radius of circle $= \frac{1}{2} \times (114.08) \times 7$

$= 399.28\ cm^2$

CASE BASED QUESTIONS(4 MARKS QUESTIONS)

(1) A student draws two circles that touch each other externally at point K with centres A and B and radii 6 cm and 4 cm, respectively as shown in the figure.



(i)How many common tangents can be drawn, if two circles touch externally
(ii)Find the length of XY
(iii)(a)Find the sum of the areas of two circles (use $\pi = 3.14$)

(OR)

(b)Find the length of PQ

Ans) (i) 3 tangents

(ii)20 cm

(iii)(a) Sum of the areas = $\pi(R^2 + r^2) = 3.14(36 + 16) = 3.14 X 52 = 163.28\ cm^2$

OR

(b) PA = 10 cm and QB = 5 cm (by Pythagoras theorem)

PQ = PA + AB + QB = 10 + 10 + 5 = 25 cm

(2) The picture given below shows a circular mirror hanging on the wall with a cord. The diagram represents the mirror as a circle with centre O, NA and NB are tangents to the circle at A and B respectively such that NA = 30 cm and $\angle ANB = 60^0$.

Based on above information answer the following

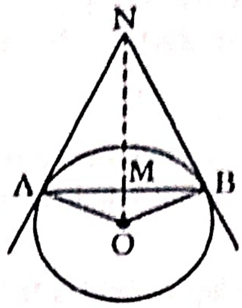
(i)Find the length of AB

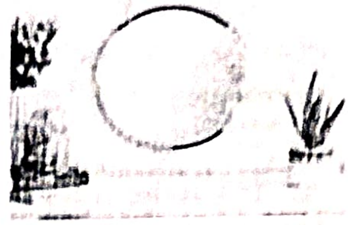
(ii)Find $m\angle AOB$

(iii)(a) Find the length of ON

(OR)

(a) Find the radius of the mirror.





Ans) (i) In rt $\Delta AMN, \angle ANM = 30^0$

$\Rightarrow \tan 30^0 = \frac{AM}{AN} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AM}{30} \Rightarrow AM = 10\sqrt{3}\ cm$ and AB = $20\sqrt{3}$ cm.

(ii) $\angle AOB = 120^0$

(iii)(a) $cos30^0 = \frac{AN}{ON} = \frac{\sqrt{3}}{2} = \frac{30}{ON}$

$ON = \frac{60}{\sqrt{3}} = 20\sqrt{3}$ cm

(OR)

(b) $OA^2 = ON^2 - AN^2 = 1200 - 900 = 300 \Rightarrow OA = 10\sqrt{3}\ cm$

(3) The discus throw is an event in which an athlete attempts to throw a discus. The athlete spins anti-clockwise around one and a half times through a circle, then releases the throw. When released, the discus travels along tangent to the circular spin orbit.

In the given figure, AB is one such tangent to a circle of radius 75 cm.

Point O is centre of the circle and ∠ABO = 30°. PQ is parallel to OA

Based on the above, information:

(i)Find the length of AB.

(ii) Find the length of OB.

(iii) (a) Find the length of AP.

(OR)

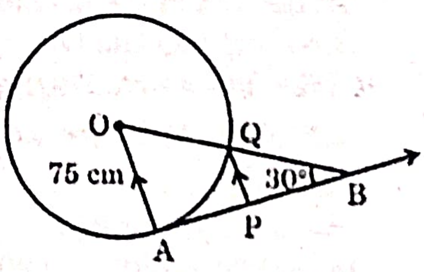
(iii)(b) Find the length of PQ.

Ans: (i) use tan 30^0 ratio then AB = $75\sqrt{3}$ cm

(ii) use sin 30^0 ratio then OB = 150 cm

(iii)(a) QB = 150 – 75 = 75 cm ⇒ Q is mid point. of OB Since PQ || AO therefore P is mid point of AB

AP = $37.5\sqrt{3}$ cm



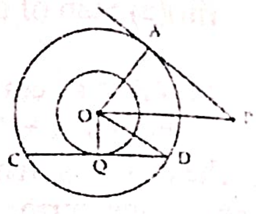
(iii)(b) QB = 150 – 75 = 75 cm.

$\Delta BQP \sim \Delta BOA (\,by\ AA\ similarity\,) \Rightarrow \frac{QB}{OB} = \frac{PQ}{AO}$

$\Rightarrow \frac{75}{150} = \frac{PQ}{75} \Rightarrow PQ = \frac{75}{2}$ cm .

HOTS(HIGH ORDER THINKING SKILLS)

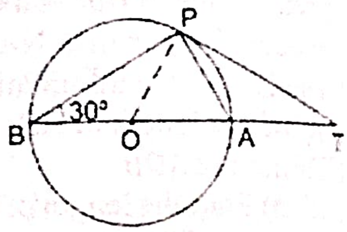
(1) In two concentric circles, the radii are OA= r cm and OQ=6 cm, as shown in the figure. Chord *CD* of larger circle is a tangent to smaller circle at Q. PA is tangent to larger circle. If *PA*=16 cm and *OP*=20 cm, find the length *CD*.



Ans) $r^2 = 20^2 - 16^2 = 144 \Rightarrow r = 12\ cm \Rightarrow OD = r = 12\ cm$

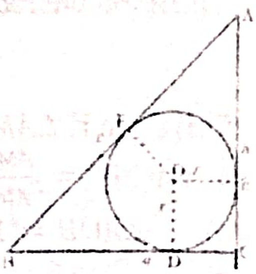
$DQ^2 = OD^2 - OQ^2 = 144 - 36 = 108 \Rightarrow DQ = 6\sqrt{3}$ cm $\Rightarrow CD = 12\sqrt{3}$ cm .

(2) In the following Fig, BOA is a diameter of a circle and the tangent at a point P meets BA extended at T. If $\angle PBO = 30°$, then find $\angle PTA$.



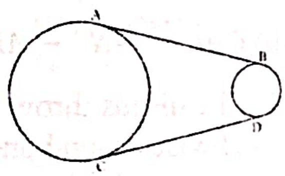
Ans) As $\angle BPA = 90°$, $\angle OBP = \angle OPB = 30°$, $\angle PAB = \angle OPA = 60°$.
Also, OP $\perp$ PT. Therefore, $\angle APT = 30°$ and $\angle PTA = 60° - 30° = 30°$.

(3)If a, b, c are the sides of a right triangle where c is the hypotenuse, prove that the radius r of the circle which touches the sides of the triangle is given by $r = \frac{a+b-c}{2}$

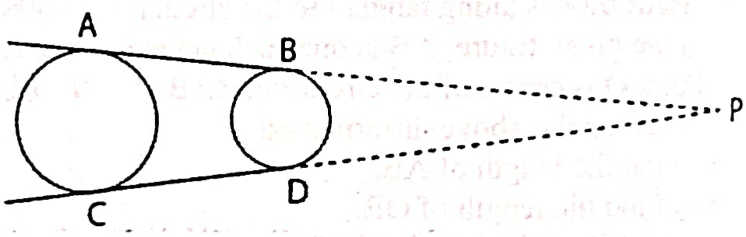


Ans) : Let the circle touches the sides BC, CA, AB of the right triangle ABC at D, E and F respectively,
where BC = a, CA = b and AB = c (see Fig.). Then AE = AF and BD = BF.
Also CE = CD = r.[OECD is a square]
i.e., b – r = AF, a – r = BF or AB = c = AF + BF = b – r + a – r
$\Rightarrow c = a + b - 2r \Rightarrow 2r = a + b - c$
This gives $r = \frac{a+b-c}{2}$

(4) In the following Fig. AB and CD are common tangents to two circles of unequal radii. Prove that AB = CD.

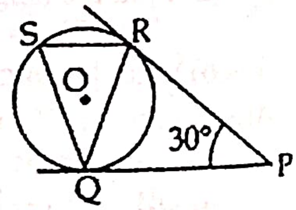


Ans) Produce AB and CD to intersect at P.



PA = PC and PB = PD
$\Rightarrow PA - PB = PC - PD \Rightarrow AB = CD$

(5) In the figure, tangents PQ and PR are drawn from an external point P to a circle with centre O, such that $\angle RPQ = 30°$. A chord RS is drawn parallel to the tangent PQ. Find $\angle RQS$.



Ans) Join OQ and OR
PR = PQ $\Rightarrow In\ \Delta PQR,\ \angle PRQ = \angle PQR = 75^0$
$\angle ROQ = 150^0 \Rightarrow \angle QSR = \frac{1}{2}\angle QOR = 75^0$
SR $\parallel$ PQ
$\Rightarrow \angle SRQ = \angle PQR = 75^0$ (*alt.int.angles, QR is a transversal*)
In $\Delta QSR, \angle RQS = 180^0 - (\angle QSR + \angle SRQ) = 180^0 - 150^0 = 30^0$

$$\angle RQS = 30^0$$